M A T H E M A T I C S

(NDA & JEE MAINS)

Definitions (Phase - II) Introduction

* **Numbers are basically categorised into two groups**
  1. **Real numbers** 2. **Complex numbers**
* **Real Number:** Any number which is not complex is called a real number. Real numbers are denoted by 'R'. Set of real numbers is given by R = (Q, IR).
* **Complex number** : Any number which is in the form of n + ib (or) x + iy, where the value of 'i' is $\sqrt{-1}$. i is called iota. this is called a complex number. Complex numbers denoted by 'Z'

  Ex: $\sqrt{-2}, \sqrt{-3}, \sqrt{-4}$, where

  $\sqrt{-4} = \sqrt{4} \times -1$

  $= \sqrt{4} \times \sqrt{-1}$

  $= 2\sqrt{-1}$

  $= 2i$
* **Rational number** : Any number which has numerator and denominator is called a rational number. Rational numbers are denoted by 'Q'

  (or)

  Any number which is in the form of p/q, q ≠ 0 is called a rational number.

  Ex : 1, 2, 5/2, 3/3, 6/2
* **Pure Rational Number** : Any number which is in the form of p/q, where q ≠ 0 and p, q being 'Coprimes' is called pure rational (or) rational number.

  (Or)

  Any number which is in the form of p/q, q ≠ 0, 1 and p, q being 'Coprimes' is called pure rational number.

  Ex : 1/2, 5/2*2, 3/4
* **Irrational number** : Any number which is not rational is called an irrational number. Irrational numbers are also called surds. Irrational numbers are denoted by 'IR'.

  Ex : $\sqrt{2}, \sqrt{3}, 2\sqrt{2}, \sqrt{3}$, ...

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* **Natural numbers set** : A set of numbers which starts from '1' and ends with infinity is called a natural numbers set. Natural numbers are denoted by 'N'.
  N = {1, 2, 3, 4, ...n}
* **Whole numbers set** : A set of numbers which start from '0' and ends with infinity is called a whole numbers set. Whole numbers are denoted by 'W'.
  W = {0, 1, 2, 3, ...n}
* **Integers:** A set of numbers which starts from negative (minus) infinity and ends with positive (plus) infinity is called set of integers. Intergers are denoted by 'I'.
  I = {-2, -1, 0, 1, 2, ...n}
* **Proper fraction**: A fraction in which numerator is less than the denominator is called a proper fraction.
  Ex: 1/2, 1/3, 1/8, 7/7
* **Improper fraction**: A fraction in which numberator is greater than the denominator is called improper fraction.
  Ex: 25/6, 4/3, 5/5
* **Like fractions:** Fractions whose denominators are same are called like fractions.
  Ex: 1/2, 3/2, 4/2
* **Unlike fractions:** Fractions whose denominators are not same are called unlike fractions.
  Ex: 1/2, 3/4, 5/6, ...
* **Mixed fractions**: A fraction which can be expressed as a whole part and a fraction part is called a mixed fraction.
  Ex: 2 1/2, 5 3/4
* **Decimal form of a Rational number**: Any rational number which is expressed in the decimal form is called the decimal form of a rational number.
  Ex: (i) The decimal form of 5/2 is 2:5
  (ii) The decimal form of 1/3 is 0.3333...
* **Terminating decimal expansion**: A decimal form in which the digits terminates (end) immediately after the decimal point or few digits is called a

terminating decimal expansion.
Ex: 1.5, 2.35, 3.345...

* Non Terminating decimal expansion: A decimal form in which the digits doesn't terminate (end) immediately after the decimal point or after few digits is called a non- terminating decimal expansion.

Ex: 0.33333...1.43467...

Note:

* There are two types of Non-terminating decimal expansions.
(i) Non terminating repeating decimal expansion
(ii) Non terminating non repeating decimal expansion
* Non Terminating repeating decimal expansion : A non terminating decimal expansion in which the digit or digits repeats immediately after the decimal point or after few digits is called non-terminating Recurring decimal expansion.

Eg : 1.333, 1.2566.....

Note: There are two types of Non terminating repeating decimal expansions

* Non-Terminating impure repeating decimal expansion : A non terminating repeating decimal expansion in which the digit or digits repeats after few digits of the decimal point is called a non-terminating impure repeating decimal expansion.

Eg: 1.25666.....

* Non-Terminating pure repeating decimal expansion : A non-Terminating repeating decimal expansion in which the digit or digits repeats immediately after the decimal point is called a Non-Terminating pure repeating decimal expansion.

Eg: 1.3333.....

Note: (i) Period: The digit or digits on which the bar is present is called a period.
(ii) Periodicity: The total number of digits (or digit) on which the bar is present is called a periodicity.

Example: (i) 2.352 period - 52, periodicity - 2.
(ii) 0.612 period - 612, periodicity - 3.

* **Non-Terminating non repeating decimal expansion** : A decimal form in which the digits after the decimal point does not terminate (end) and does not repeat is called Non-terminating non Repeating decimal expansion.

Eg: 1.435689.....

Note:

* (i) Condition for a rational number (p/q) to be a terminating decimal expansion : If the denominator of any rational number is expressed or converted in the form of 2^m * 5^n, then the rational number is said to be a terminating decimal expansion.

Example: (i) $\frac{2}{125} = \frac{2}{2^0 \times 5^3}$ (ii) $\frac{1}{16} = \frac{1}{2^4 \times 5^0}$

(ii) Condition for a rational number (p/q) to be a Non-terminating decimal expansion : If the denominator of any rational number is not expressed or converted in the form of 2^m * 5^n, then the rational number is said to be a Non-terminating decimal expansion.

Example: (i) $\frac{1}{150} = \frac{1}{2^1 \times 3^1 \times 5^2}$ (ii) $\frac{2}{75} = \frac{2}{3^1 \times 5^2}$

* **Even number**: Any number which is divisible by 2 is called an even number.

Ex: 2, 4, 6, 8, 10

Note:

* The least even number is 2
* The standard or the general form of any even number is 2n.
* $even^{even} = even$
* even + even = even
* even - even = even
* even x even = even
* $even^{odd} = even$
* even - odd = odd
* Odd Number: Any number which is not divisible by 2 is called an odd number

Eg: 1, 3, 5, 7, 9, 11

Note:

* The least odd number is 1
* The standard form of an odd number is 2n - 1.

* $odd^{odd} = odd$
* odd + odd = even
* odd - odd = even
* $odd^{evem} = odd$
* $even^{even} = even$
* odd / even = odd or even / odd = odd
* odd - even = even - odd = odd
* odd x odd = odd
* odd x even = even x odd = even
* **Prime number**: A number which has exactly two factors and the factors being "1" and "itself" is called a prime number.

  Example: 2, 3, 5, 7, 11, 13, 17

  Note:
* The least prime number is 2.
* 2 is the only even prime number and the remaining all are odd numbers.
* There are 9 prime numbers between 1 to 25, 6 prime numbers between 25 to 50, 10 prime numbers between 50 to 100, 15 prime numbers between 1 to 50 and 25 prime numbers between 1 to 100.
* The prime numbers between 1 to 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
* **Prime triplet**: The set of three consecutive primes is called a prime triplet.

  Example: {3, 5, 7}, {2, 3, 5}
* **Composite number**: A number which has three or more than three favors is called a composite number.

  Ex: 4, 6, 8, 9, 10, 12

  Note:
* The least composite number is '4'.
* '1' is neither prime nor composite.
* **Co-primes [Relatively Primes]**: If the HCF of any two numbers is '1', then the two numbers are called co-primes.

  Example: (i) 5, 8 (ii) 15, 8

  Note:
* Any two primes are always co-primes.
* **Twin Primes:** If the difference between any two prime numbers is '2', then the two numbers are called twin prime numbers.

Example: (i) 3, 5 (ii) 5, 7 (iii) 11, 13

Note:

* All twin primes are co-primes
* **Perfect Number:** A number for which the sum of all its factors is equal to twice the number is called a perfect number.

  Example: (i) 6 - $(1 + 2 + 3 + 6 = 12)$

  (ii) 28 - $(1 + 2 + 4 + 7 + 14 + 28 = 56)$

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'LAWS OF RADICALS'

1. $a^n \times a^m = a^{n+m}$
2. $a^n \times a^n \times a^n = a^{n+n+n}$
3. $a^n \times a^n \times a^n \times \ldots\ldots = a^{n+n+n+}\ldots\times$
4. $a^{\,n}/^n = a^1$
5. $(a^n)^m = a^{nm} = a^{mn}$
6. $[(a^n)^m]^p = a^{nmp} = a^{nmp} = a^{nmp}$
7. $a^{-n} = 1/a^n$
8. $1/a^n = a^{-n}$
9. $(ab)^n = a^n \times b^n$
10. $(a/b)^n = a^n/b^n$
11. If $a^n = b^n$ then $a = b$.
12. If $a^m = a^n$ then $m = n$
13. $a^0 = 1$
14. $(a/b)^{-n} = (b/a)^n$

Algebraic Identities

1. $(a + b)(a - b)$ or $(a - b)(a + b) = a^2 - b^2$.
2. $(a + b)^2$ or $(b + a)^2 = a^2 + 2ab + b^2$
3. $(a - b)^2$ or $(b - a)^2 = a^2 - 2ab + b^2$
4. $a^2 + b^2 = (a + b)^2 - 2ab$
5. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
6. $(a + b)^2 - (a - b)^2 = 4ab$
7. $(a + b)^3 = a^3 + 3ab(a + b) + b^3$ or $a^3 + 3a^2b + 3ab^2 + b^3$
8. $(a - b)^3 = a^3 - 3ab(a - b) - b^3$ or $a^3 - 3a^2b + 3ab^2 - b^3$
9. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ or $(a + b)^3 - 3ab(a + b)$
10. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ or $(a - b)^3 + 3ab(a - b)$
11. $(a + b + c)^2$ or $(-a - b - c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ or $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
12. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
    If $a + b + c = 0$ or $a^2 + b^2 + c^2 - ab - bc - ca = 0$, then
    $a^3 + b^3 + c^3 - 3abc = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$
13. $(x + a)(x + b) = x^2 + (a + b)x + ab$
14. $(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

LOYAL EDUCATION
Result Oriented

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Points to remember

1. Additive Identity of any number is "0" (zero).
2. Multiplicative Identity of any number is "1".
3. Additive Inverse: If the sum of any two numbers is zero, then the two numbers are additive inverse of each other.
4. Multiplicative Inverse: If the product of any two numbers is one, then the two number are multiplicative inverse of each other.
5. If the unit digit of a number is one among 0, 2, 6, 8 (even), then the number is divisible by 2.
6. If the sum of the digits of a number is divisible by 3, then the number is divisible by 3.
7. If the last two digits of a number is divisible is 4, then the number is divisible by 4.
8. If a number is divisible by 2, 3 both, then the number is divisible by 6.
9. If the unit digit of any number is either "0" or "5", then the number is divisible by 5.
10. If the last three digits of any number are divisible by 8, then the number is divisible by 8.
11. If the sum of the digits of a number is divisible by 9, then the number is divisible by 9.
12. If the unit digit of a number is 0, then the number is divisible by 10.
13. If the difference of the sum of the digits on the alternate places is divisible by 11, then the number is divisible by 11.
14. I - 1
15. V - 5
16. X - 10
17. L - 50
18. C - 100
19. D - 500
20. M - 1000
21. Closure Property: $a + b = c$, $ab = c$
22. Associative Property: $a + (b + c) = (a + b) + c$, $a(bc) = (ab)c$
23. Commutative property: $a + b = b + a$, $ab = ba$
24. Distributive property: $a (b + c) = ab + ac$

25. The standard form of a natural number (or) a whole number (or) an integer is 'x' (or) 'n'.
26. The standard form of a rational number p/q (or) a/b.
27. The standard of a real number is $a = bq + r$, where $0 \leq r < b$ and 'a' is called the dividend, 'b' is called the divisor, 'q' is called the quotient and 'r' is called the remainder. This is also called Division rule (or) Euclid's division rule.
28. The standard form of an even number is 2x (or) 2n.
29. The standard form of an odd number is 2x + 1 (or) 2n +1.
30. The standard form of two consecutive natural numbers (or) whole number (or) integers is x, x+1
31. The standard form of a Linear equation in one variable is ax+b=0
32. The standard form of a Linear equation in two variables is ax + by + c=0
33. The standard form of a Quadratic equation is $ax^2 + by + c = 0$
34. Perimeter of semicircle -
35. Perimeter of quadrant of a circle =
36. Area of circle = πr^2
37. Perimeter of circle = $2\pi r$
38. Area of semicircle =
39. Area of quadrant of a circle =
40. Area of scalene triangle = $\sqrt{(s(s - a) (s - b) (s - c)}$
41. Area of quadrilateral = $\frac{1}{2}$ $d(h_1 + h_2)$
42. Area of isosceles triangle = $\frac{1}{2}$ bh
43. Area of right isosceles triangle =
44. Area of equilateral triangle = $\sqrt{3} / 4\ a^2$
45. Height of equilateral triangle = $\sqrt{3} / 2$ a
46. Area of trapezium = $\frac{1}{2}$ h(a + b)
47. Area of Rhombus = $\frac{1}{2}$ d_1 x d_2
48. Point of intersection of all the 3 medians of a triangle is called a Centroid (G)
49. Point of intersection of all the 3 perpendiculars of a triangle is called a Orthocentre (O).
50. Point of intersection of all the 3 perpendicular bisectors of a triangle is called a Circumcentre (S).
51. Point of intersection of all the 3 angle bisectors (internal) of a triangle is called a Incentre (I).

52. Point of intersection of all the 3 angle bisectors (external) of a triangle is called a Excentre (Ex).

Squares	Cubes	Squares of prime numbers	Square root values
$1^2 = 1$	$1^3 = 1$	$2^2 = 4$	$\sqrt{2} = 1.41421356$
$2^2 = 4$	$2^3 = 8$	$3^2 = 9$	$\sqrt{3} = 1.73205081$
$3^2 = 9$	$3^3 = 27$	$5^2 = 25$	$\sqrt{5} = 2.23606797$
$4^2 = 16$	$4^3 = 64$	$7^2 = 49$	$\sqrt{7} = 2.64575131$
$5^2 = 25$	$5^3 = 125$	$11^2 = 121$	$\sqrt{11} = 3.31662479$
$6^2 = 36$	$6^3 = 216$	$13^2 = 169$	
$7^2 = 49$	$7^3 = 343$	$17^2 = 289$	
$8^2 = 64$	$8^3 = 512$	$19^2 = 361$	
$9^2 = 81$	$9^3 = 729$	$23^2 = 529$	
$10^2 = 100$	$10^3 = 1000$	$29^2 = 841$	
$11^2 = 121$	$11^3 = 1331$	$31^2 = 961$	
$12^2 = 144$	$12^3 = 1728$	$37^2 = 1369$	
$13^2 = 169$	$13^3 = 2197$	$41^2 = 1681$	
$14^2 = 196$	$14^3 = 2744$	$43^2 = 1849$	
$15^2 = 225$	$15^3 = 3375$	$47^2 = 2209$	
$16^2 = 256$	$16^3 = 4096$	$53^2 = 2809$	
$17^2 = 289$	$17^3 = 4913$	$59^2 = 3481$	
$18^2 = 324$	$18^3 = 5832$	$61^2 = 3721$	
$19^2 = 361$	$19^3 = 6859$	$67^2 = 4489$	
$20^2 = 400$	$20^3 = 8000$	$71^2 = 5041$	
$21^2 = 441$		$73^2 = 5329$	
$22^2 = 484$		$79^2 = 6241$	
$23^2 = 529$		$83^2 = 6889$	
$24^2 = 576$		$89^2 = 7921$	
$25^2 = 625$		$97^2 = 9409$	
$26^2 = 676$			
$27^2 = 729$			
$28^2 = 784$			
$29^2 = 841$			
$30^2 = 900$			